27.04.2023 TUTORIAL - II

1. A farmer wishes to test the effects of four different fertilizers A, B, C, D on the yield of wheat. In order to eliminate sources of error due to variability in soil fertility, he uses the fertilizers, in a Latin square arrangement as indicated in the following table, where the numbers indicate yields in bushels per unit area.

A 18	C 21	D 25	B 11
D 22	B 12	A 15	C 19
B 15	A 20	C 23	D 24
C 22	D 21	B 10	A 17

Perform an analysis of variance to determine, if there is a significant difference between the fertilizers at $\alpha = 0.05$ level of significance.

Sol: Origin : 15.

	X_1	X_2	X_3	X_4	Total	X_1^2	X_2^2	X_3^2	X_4^2
Y_1	A 3	C 6	D 10	B -4	15	9	36	100	16
Y_2	D 7	B -3	A 0	C 4	8	49	9	0	16
Y_3	B 0	A 5	C 8	D 9	22	0	25	64	81
Y_4	C 7	D 6	B -5	A 2	10	49	36	25	4
Total	17	14	13	11	55	107	106	189	117

H_0 : There's no significant difference between column means, row means and treatements.

H_1 : There's significant difference between column means, row means and treatments.

Step 1 : $N = 16$

Step 2 : $T = 55$

Step 3 : $\frac{T^2}{N} = \frac{3025}{16} = 189.06$

Step 4 : $TSS = \Sigma x_1^2 + \Sigma x_2^2 + \Sigma x_3^2 + \Sigma x_4^2 - \frac{T^2}{N}$

$= 107 + 106 + 189 + 117 - 189.06$

$TSS = 329.94$

Step 5: $SSC = \frac{(\Sigma X_1)^2}{N_1} + \frac{(\Sigma X_2)^2}{N_1} + \frac{(\Sigma X_3)^2}{N_1} + \frac{(\Sigma X_4)^2}{N_1} - \frac{T^2}{N}$

$= \frac{289}{4} + \frac{196}{4} + \frac{169}{4} + \frac{121}{4} - 189.06$

$= 72.25 + 49 + 42.25 + 30.25 - 189.06$

$= 193.75 - 189.06$

$SSC = 4.69$

Step 6: $SSR = \frac{(\Sigma Y_1)^2}{N_2} + \frac{(\Sigma Y_2)^2}{N_2} + \frac{(\Sigma Y_3)^2}{N_2} + \frac{(\Sigma Y_4)^2}{N_2} - \frac{T^2}{N}$

$= \frac{225}{4} + \frac{64}{4} + \frac{484}{4} + \frac{100}{4} - 189.06$

$= 56.25 + 16 + 121 + 25 - 189.06$

$= 218.25 - 189.06$

$SSR = 29.19$

Step 7: SSK

Arrange the elements on the order of treatments To find SSK.

					Total
A	3	0	5	2	10
B	-4	-3	0	-5	-12
C	6	4	8	7	25
D	10	7	9	6	32

$SSK = \frac{(10)^2}{4} + \frac{(-12)^2}{4} + \frac{(25)^2}{4} + \frac{(32)^2}{4} - 189.06$

$= \frac{100}{4} + \frac{144}{4} + \frac{625}{4} + \frac{1024}{4} - 189.06$

$= 25 + 36 + 156.25 + 256 - 189.06$

$= 473.25 - 189.06$

$= 284.19$

Step 8:

$SSE = TSS - SSC - SSR - SSK.$

$= 329.94 - 4.69 - 29.19 - 284.19$

$= 11.87$

Step 9 : ANOVA table

Source of variation	Sum of Squares	DOF	Mean of Squares	Variance ratio	Table value
Between columns	$SSC = 4.69$	$k-1$ $= 4-1$ $= 3$	$MSC = \frac{SSC}{k-1}$ $= \frac{4.69}{3}$ $MSC = 1.56$	$F_C = \frac{MSE}{MSC}$ $= \frac{1.98}{1.56}$ $F_C = 1.27$	$F_C(6,3)$ $= 8.94$
Between rows	$SSR = 29.19$	$k-1$ $= 4-1$ $= 3$	$MSR = \frac{SSR}{k-1}$ $= \frac{29.19}{3}$ $= 9.73$	$F_R = \frac{MSR}{MSE}$ $= \frac{9.73}{1.98}$ $F_R = 4.91$	$F_C(3,6)$ $= 4.76$
Between treatments	SSK $= 284.19$	$k-1$ $= 3$	$MSK = \frac{SSK}{k-1}$ $= \frac{284.19}{3}$ $= 94.73$	$F_T = \frac{MSK}{MSE}$ $= \frac{94.73}{1.98}$ $F_T = 47.84$	$F_T(3,6)$ $= 4.76$
Error	SSE $= 11.87$	$(k-1)(k-2)$ $= 3 \times 2$ $= 6$	$MSE = \frac{SSE}{(k-1)(k-2)}$ $= \frac{11.87}{6}$ $= 1.98$		

Step 10 : Conclusion

Cal F_C < Tab F_C ; So we accept H_0.

Cal F_R > Tab F_R ; So we reject H_0

Cal F_T > Tab F_T ; So we reject H_0.

27.04.2023

TUTORIAL - 12.

1. What are the basic principles of the design of experiments?

There are 3 basic principles of experimental design, they are

* Randomization
* Replication
* Local control

* Randomization

* A set of objects ~~set~~ is said to be randomized, when they are arranged in random order.

* The most frequently used assumption is the one which relates the observations (units) are independent.

* Replication.

* The independent execution of an experiment more than once is called replication.

* It is necessary to increase the accuracy of estimates of the treatement effects.

* Local control.

This includes techniques such as grouping, blocking and balancing of experimental units used in a experimental design.

2. What are the assumptions involved in ANOVA?

* The observations are independent.

* Parent populations from which observations are taken is normal.

* Various treatment and environmental effects are additive, in nature.

3. Why a 2×2 Latin Square is not possible? Explain.

Consider a $n \times n$ Latin Square design, then the degrees of freedom for SSE is

$$= (n^2 - 1) - (n-1) - (n-1) - (n-1)$$

$$= n^2 - 1 - 3n + 3$$

$$= n^2 - 3n + 2$$

$$= (n-1)(n-2).$$

$$2 < \begin{matrix} -2 \\ -1 \\ \hline -3 \end{matrix}$$

For $n = 2$, df of SSE $= 0$

Hence MSE is not defined.

∴ Comparisons are not possible. Hence, a 2×2 Latin Square Design is not possible.

4. Present the ANOVA table for one way classification

Source of variation	Sum of Squares	Degrees of freedom	Mean squares	Variance ratio	Table value
Between columns	SSC	$C-1$	$MSC = \frac{SSC}{C-1}$	$F_C = \frac{MSC}{MSE}$ or $F_C = \frac{MSE}{MSC}$	
Error	SSE	$N-C$	$MSE = \frac{SSE}{N-C}$		

5. Present the ANOVA table for two-way classification.

Source of variation	Sum of Squares	D.f	Mean Squares	Variance ratio	Table value
Between columns.	SSC	$c-1$	$MSC = \frac{SSC}{c-1}$	$F_C = \frac{MSC}{MSE}$ or $F_C = \frac{MSE}{MSC}$	

Between rows	SSR	$r-1$	$MSR = \frac{SSR}{r-1}$	$F_R = \frac{MSR}{MSE}$ or $\frac{MSE}{MSR}$
Error	SSE	$(c-1)(r-1)$	$MSE = \frac{SSE}{(c-1)(r-1)}$	

6. Compare and contrast Latin Square Design & RBD.

LSD	RBD
* Variation is controlled in two directions.	* Variation is controlled in one direction only.
* Experimental area must be a square.	* Suitable, if it is a rectangle or square.
* The number of rows and columns are equal	* There is no such restrictions.
* The number of replication is equal to the number of treatments.	* It can have any number replications and treatments.
* It is suitable for small number of treatments, between 5 and 12.	* No such restrictions suitable for upto 24 treatements.

7. What are the advantages of the Latin Square design over other designs.

* With two-way classification, the Latin Square controls more of the variation than the CRR or RBD. The two-way elimination of variation often results in small error mean square.

* The analysis is simple, it is only slightly more complicated than that for the randomized complete block design.

* The analysis remains relatively simple even with missing data. Analytical procedures are available for omitting one or more treatements, rows or columns.

8. Write down the ANOVA table for Latin Square design.

Source of variation	Sum of squares	d.f	Mean Squares	Variance ratio	Table value
Between columns	SSC	k-1	$MSC = \frac{SSC}{k-1}$	$F_c = \frac{MSC}{MSE}$ or $\frac{MSE}{MSC}$	
Between rows	SSR	k-1	$MSR = \frac{SSR}{k-1}$	$F_R = \frac{MSR}{MSE}$ or $\frac{MSE}{MSR}$	
Between Treatments	SSK	k-1	$MSK = \frac{SSK}{k-1}$	$F_K = \frac{MSK}{MSE}$ or $\frac{MSE}{MSK}$	
Error	SSE	(k-1)(k-2)	$MSE = \frac{SSE}{(k-1)(k-2)}$		

9. What are the uses of Analysis of variance (ANOVA)?

* Analysis of Variance (ANOVA) is a statistical formula used to compare variances across the means of different groups.

* It helps to find out the F-test.

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* To test the homogeneity of several means.